



**SOUTHERN PLAINS**  
TRANSPORTATION CENTER

## **Development of a Numerical Simulation Tool for Continuously Reinforced Concrete Pavements**

Cesar Carrasco, Ph.D.  
Soheil Nazarian, Ph.D., P.E.  
Nancy Aguirre

**SPTC14.1-94-F**

**Southern Plains Transportation Center  
201 Stephenson Parkway, Suite 4200  
The University of Oklahoma  
Norman, Oklahoma 73019**

## ***DISCLAIMER***

*The contents of this report reflect the views of the authors, who are responsible for the facts and accuracy of the information presented herein. This document is disseminated under the sponsorship of the Department of Transportation University Transportation Centers Program, in the interest of information exchange. The U.S. Government assumes no liability for the contents or use thereof.*

## Technical Report Documentation Page (TRDP)

1. REPORT NO. <b>SPTC14.1-94</b>	2. GOVERNMENT ACCESSION NO.	3. RECIPIENTS CATALOG NO.	
4. TITLE AND SUBTITLE <b>Development of a Numerical Simulation Tool for Continuously Reinforced Concrete Pavements</b>		5. REPORT DATE <b>November 20, 2018</b>	
7. AUTHOR(S) I. Cesar Carrasco II. Soheil Nazarian		6. PERFORMING ORGANIZATION CODE	
9. PERFORMING ORGANIZATION NAME AND ADDRESS <b>The University of Texas at El Paso 500 W. University Ave. El Paso, Texas 79968</b>		8. PERFORMING ORGANIZATION REPORT	
12. SPONSORING AGENCY NAME AND ADDRESS <b>Southern Plains Transportation Center 201 Stephenson Pkwy, Suite 4200 The University of Oklahoma Norman, OK 73019</b>		10. WORK UNIT NO.	
15. SUPPLEMENTARY NOTES <b>University Transportation Center</b>		11. CONTRACT OR GRANT NO. <b>DTRT13-G-UTC36</b>	
16. ABSTRACT		13. TYPE OF REPORT AND PERIOD COVERED <b>Final September 2015 – September 2018</b>	
17. KEY WORDS <b>Continuously Reinforced Concrete Pavement (CRCP), mechanistic-empirical pavement design, Finite Element Method (FEM)</b>		14. SPONSORING AGENCY CODE	
19. SECURITY CLASSIF. (OF THIS REPORT) <b>Unclassified</b>		18. DISTRIBUTION STATEMENT <b>No restrictions. This publication is available at <a href="http://www.sptc.org">www.sptc.org</a> and from the NTIS.</b>	
20. SECURITY CLASSIF. (OF THIS PAGE) <b>Unclassified</b>		21. NO. OF PAGES <b>46</b>	22. PRICE

# Metric Conversion Page

<b>SI* (MODERN METRIC) CONVERSION FACTORS</b>				
<b>APPROXIMATE CONVERSIONS TO SI UNITS</b>				
<b>SYMBOL</b>	<b>WHEN YOU KNOW</b>	<b>MULTIPLY BY</b>	<b>TO FIND</b>	<b>SYMBOL</b>
<b>LENGTH</b>				
in	inches	25.4	millimeters	mm
ft	feet	0.305	meters	m
yd	yards	0.914	meters	m
mi	miles	1.61	kilometers	km
<b>AREA</b>				
in <sup>2</sup>	square inches	645.2	square millimeters	mm <sup>2</sup>
ft <sup>2</sup>	square feet	0.093	square meters	m <sup>2</sup>
yd <sup>2</sup>	square yard	0.836	square meters	m <sup>2</sup>
ac	acres	0.405	hectares	ha
mi <sup>2</sup>	square miles	2.59	square kilometers	km <sup>2</sup>
<b>VOLUME</b>				
fl	fluid ounces	29.57	milliliters	mL
oz	gallons	3.785	liters	L
gal	cubic feet	0.028	cubic meters	m <sup>3</sup>
ft <sup>3</sup>	cubic yards	0.765	cubic	m <sup>3</sup>
<b>MASS</b>				
oz	ounces	28.35	grams	g
lb	pounds	0.454	kilograms	kg
T	short tons (2000 lb)	0.907	megagrams (or "metric ton")	Mg (or "t")
<b>TEMPERATURE (exact degrees)</b>				
°F	Fahrenheit	5 (F-32)/9	Celsius	°C
<b>ILLUMINATION</b>				
fc	foot-candles	10.76	lux	lx
fl	foot-Lamberts	3.426	candela/m <sup>2</sup>	cd/m <sup>2</sup>
<b>FORCE and PRESSURE or STRESS</b>				
lbf	poundforce	4.45	newtons	N
lbf/in <sup>2</sup>	poundforce per square inch	6.89	kilopascals	kPa
<b>APPROXIMATE CONVERSIONS FROM SI UNITS</b>				
<b>SYMBOL</b>	<b>WHEN YOU KNOW</b>	<b>MULTIPLY BY</b>	<b>TO FIND</b>	<b>SYMBOL</b>
<b>LENGTH</b>				
mm	millimeters	0.039	inches	in
m	meters	3.28	feet	ft
m	meters	1.09	yards	yd
km	kilometers	0.621	miles	mi
<b>AREA</b>				
mm <sup>2</sup>	square millimeters	0.0016	square inches	in <sup>2</sup>
m <sup>2</sup>	square meters	10.764	square feet	ft <sup>2</sup>
m <sup>2</sup>	square meters	1.195	square yards	yd <sup>2</sup>
ha	hectares	2.47	acres	ac
km <sup>2</sup>	square kilometers	0.386	square miles	mi <sup>2</sup>
<b>VOLUME</b>				
mL	milliliters	0.034	fluid ounces	fl oz
L	liters	0.264	gallons	gal
m <sup>3</sup>	cubic meters	35.314	cubic feet	ft <sup>3</sup>
m <sup>3</sup>	cubic meters	1.307	cubic yards	yd <sup>3</sup>
<b>MASS</b>				
g	grams	0.035	ounces	oz
kg	kilograms	2.202	pounds	lb
Mg (or "t")	megagrams (or "metric ton")	1.103	short tons (2000 lb)	T
<b>TEMPERATURE (exact degrees)</b>				
°C	Celsius	1.8C+32	Fahrenheit	°F
<b>ILLUMINATION</b>				
lx	lux	0.0929	foot-candles	fc
cd/m <sup>2</sup>	candela/m <sup>2</sup>	0.2919	foot-Lamberts	fl
<b>FORCE and PRESSURE or STRESS</b>				
N	newtons	0.225	poundforce	lbf
kPa	kilopascals	0.145	poundforce per square inch	lbf/in <sup>2</sup>

\*SI is the symbol for the International System of Units. Appropriate rounding should be made to comply with Section 4 of ASTM E380. (Revised March 2003)

**DEVELOPMENT OF A NUMERICAL SIMULATION  
TOOL FOR CONTINUOUSLY REINFORCED  
CONCRETE PAVEMENTS**

**Final Report**

**September 2018**

**Cesar Carrasco, Ph.D.  
Soheil Nazarian, Ph.D., P.E.  
Nancy Aguirre, Ph.D. Candidate**

**Southern Plains Transportation Center  
201 Stephenson Pkwy, Suite 4200  
The University of Oklahoma  
Norman, OK 73019**

# Table of Contents

Introduction .....	1
Problem Statement.....	2
Objectives.....	3
Theoretical Background .....	4
Rigid Pavement Analysis .....	4
Rigid Pavement System.....	4
Rigid Pavement Analysis Methods.....	5
Rigid Pavement Analysis Tools.....	6
Development of NYSLAB.....	8
Mechanistic Modeling of CRCP .....	11
Development of Mathematical Models .....	13
Asses Modeling Needs.....	13
Condition NYSLAB to Support Modeling Needs.....	22
Development of Finite Element Model of CRCP.....	29
Implementation of the CRCP Model.....	30
Parametric Studies.....	31
Implementation/Technology Transfer (if applicable).....	33
References.....	34

## List of Figures

Figure 1. Continuously Reinforced Concrete Pavement Structure (Kim et al., 2000).....	5
Figure 2. Slab on Winkler Foundation .....	6
Figure 3. Interface Element as Modeled in NYSLAB.....	9
Figure 4. Jointed Pavement Section as Modeled in NYSLAB Using the Vlasov Model.	10
Figure 5. Bond Stress-Slip Relation between Concrete and Steel (Kim et al., 2000)....	13
Figure 6. Three – Dimensional CRCP Model .....	14
Figure 7. Bond stress-slip relation between concrete and base (Kim et al., 2000).....	15
Figure 8. Bond stress-slip relation between concrete and steel (Kim et al., 2000).....	16
Figure 9. Connector Elements along the CRCP Model .....	16
Figure 10. Boundary Conditions for the CRCP Model.....	17
Figure 11. Concrete stress distribution at the top centerline and top edge of the slab ..	19
Figure 12. Concrete stress distribution.....	20
Figure 13. Bond stress-slip relation for the connector elements.....	22
Figure 14. 27-Node Hexahedron Element.....	23
Figure 15. Unit Element Subjected to a Vertical Point Load.....	24
Figure 16. Normalized Vertical Displacement for Top Nodes.....	25
Figure 17. Cantilever Beam Model in ABAQUS .....	26
Figure 18. Deflected Cantilever Beam Model in ABAQUS .....	26
Figure 19. Comparison of Cantilever Beam Responses.....	27
Figure 20. Slab-Foundation Interface Relationship (Bhatti 2006).....	29
Figure 21. Three Dimensional Representation of the CRCP Model .....	30
Figure 22. Proposed Structure for the Analysis of Continuously Reinforced Concrete Pavements .....	32

## List of Tables

Table 1. Material Properties of CRCP Model (Kim et al., 2000) .....	14
---	----



## **Executive Summary**

The accurate modeling of the main features of continuously-reinforced concrete pavements (CRCP) is of primary importance in a mechanistic-empirical pavement design procedure. The use of the finite element (FE) method as a comprehensive tool for modeling the responses of rigid pavements, CRCP in particular, has been limited because of the complexity of calculations in modeling material nonlinear behaviors, which are difficult to describe mathematically and computationally. Significant amount of research has been conducted to improve the design of CRCPs under traffic, environmental, and thermal loads. To develop a reliable model that better represents the behavior of CRCP, a clear understanding of the design features that impact CRCP responses is essential. Researchers from the University of Texas at El Paso developed NYSLAB to analyze the response of comprehensively jointed concrete pavements (JCPs) under different geometric configurations, foundation models, temperature gradient profiles and traffic loads. This tool has the capability to analyze pavements under nonlinear thermal profiles across the thickness of the slab and capture the frictional tractions between the slab and foundation. All the complications related to appropriate discretization and modeling are handled internally by the software.

This research study aims to expand the capacity of NYSLAB by integrating a CRCP model that is capable of predicting the responses of a critical section within a CRCP pavement structure subjected to traffic and environmental loading conditions. Unlike JCPs, CRCPs use reinforcing steel rather than contraction joints for crack control. Therefore, the development of a new FE model that defines the complex interaction between the reinforcement steel and concrete as well as the slab-foundation interaction due to friction and temperature changes will be implemented into the proposed tool.

## Introduction

The first portland cement concrete pavements (PCC) were typically nine inches wide and six inches thick and were poured without joints or reinforcement. Due to traffic loading, formation level and environmental effects, these pavements began to develop random cracks, which eventually caused pavement distresses and failures. In efforts to control the development of cracks, joints were added to pavements and were placed either to guarantee no cracks or to ensure cracks only at controlled areas (Won et al., 1991). Jointed concrete pavements (JCP) are now the most commonly used type of rigid pavement nationally. However, major distresses due to traffic and environmental loads, e.g. faulting and transverse, corner and longitudinal cracking, are observed in JCPs (McCullough 1994). In 1921, the U.S. Bureau of Public Roads (predecessor to Federal Highway Administration) developed continuously reinforced concrete pavements (CRCP), a design that offered the benefit of eliminating joint distresses by incorporating continuous longitudinal reinforcement with no transverse expansions or contraction joints except at bridges or pavement ends (Pasko 1998; Plei, M., and S. Tayabji. 2012). The primary advantages of utilizing CRCPs include the improvement of the ride quality, safety, design life, and the ability to handle heavier truck loading and volumes while also decreasing the need for maintenance (Caltrans 2015). Although there are many positive attributes in the use of CRCPs, distresses are still present in this type of pavement. The most common distress type in CRCP is punchout, which can be caused by inadequate amount of steel or excessively wide or close shrinkage cracks (Zollinger *et al.*, 1999; Roesler et al., 2016). The increased use of CRCPs has resulted in continuous investigation in the behavior of such pavements and in the development of design models. The current design procedures for CRCPs involve determining the proper combination of slab thickness, concrete mixture constituents and properties, and steel reinforcement content and location. Therefore, it is important to investigate the critical responses relative to the prediction of distresses found in CRCPs so that optimum design combinations can be determined.

## ***Problem Statement***

Most of the current continuously reinforced concrete pavement design methods are still based on empirical data from the AASHTO Road Test section built in Illinois during the late 1950's. The road experiment consisted of 7 miles of two-lane pavements in the form of six loops and the test studied both portland cement concrete and asphaltic concrete pavements, as well as certain types of short-span bridges. Although the test included the analysis of concrete sections, most of those sections were jointed concrete pavements and not CRCP (AASHTO 2008). In addition, since the early 1990's, traffic loads have increased significantly and capturing the responses due to critical loads is essential in designing for CRCPs.

In contrast to the empirically-based models, the mechanistic-empirical (M-E) design combines the elements of mechanical modeling and performance observations in determining the required pavement thickness for a set of design conditions. The mechanical models are based on physical, not empirical, relationships to determine pavement critical responses, i.e tensile stresses and strains, due to truck traffic loads. Essentially, mechanistic designs have the capability of changing and adapting to new developments in pavement design by relying primarily on the mechanics of materials. For this reason, state departments of transportation have been pushing towards adopting mechanistic procedures. For example, in 2005, the Texas Department of Transportation (TxDOT) initiated a rigid pavement database project (Won et al., 2009) and by 2007, TxDOT initiated a research study to develop its own mechanistic-empirical CRCP design procedures (Won et al., 2010).

Significant amount of research has been conducted to improve the design of continuously-reinforced concrete pavements. Identifying and understanding the applicability of the current design standards and analysis tools and their respective limitations is essential for the development of a new analysis tool that significantly enhances the efficiency and capabilities of finite element based continuous concrete pavement models. Analysis tools such as CRCP-9 and CRCP-10 and the new mechanistic-empirical pavement design procedure have the capability to analyze continuous pavements under traffic, environmental, and thermal loads. CRCP-9 and 10

were developed using 2-D finite element (FE) theories to calculate stresses in concrete and steel bars due to environmental loads. A fundamental limitation of the computer programs is that it only considers a section of the pavement in the analysis of wheel load stresses and dynamic tandem axle loads. In comparison to the CRCP-10 program, this study proposes a tool that not only accommodates the analysis of a single or double axel, but also allows the simulation of any full truck axle configurations on a CRCP system and then considers the critical responses from a desired section of the full analysis. The use of FE in CRCP-9 and 10 also limits the analysis to only the concrete layer instead of all pavement layers. Similar to NYSLAB, a jointed pavement analysis tool developed by researcher from the University of Texas at El Paso (UTEP), the proposed tool will have no limit on the number of slab layers and will also provide an analysis of the supporting layers.

## ***Objectives***

A reliable prediction of pavement responses is essential for the mechanistic-empirical design procedure to evaluate the effect of environmental and traffic loads and to estimate the frequency of distresses. The objective of this study is to integrate a CRCP model into UTEP's NYSLAB analysis tool that defines the complex interaction between the reinforcement steel and concrete as well as the slab-foundation interaction due to friction and temperature changes to predict the responses of a critical section within a pavement structure subjected to traffic and environmental loading conditions. The structural model used for those predictions should consider the responses obtained from the full analysis performed in NYSLAB as input forced and should:

- Adequately describe the concrete slab properties.
- Adequately describe reinforcing steel properties (bar size and location).
- Generate a mesh dependent on the reinforcing steel arrangement.
- Adequately describe the complex bond-slip relationship between the concrete and the reinforcing steel as well as the slab-foundation interaction.
- Account for discontinuities in the pavement structure (cracks and joints).

With the ultimate purpose to predict the maximum concrete stresses, steel stresses, base deformations and subgrade strains within a critical section.

## **Theoretical Background**

### ***Rigid Pavement Analysis***

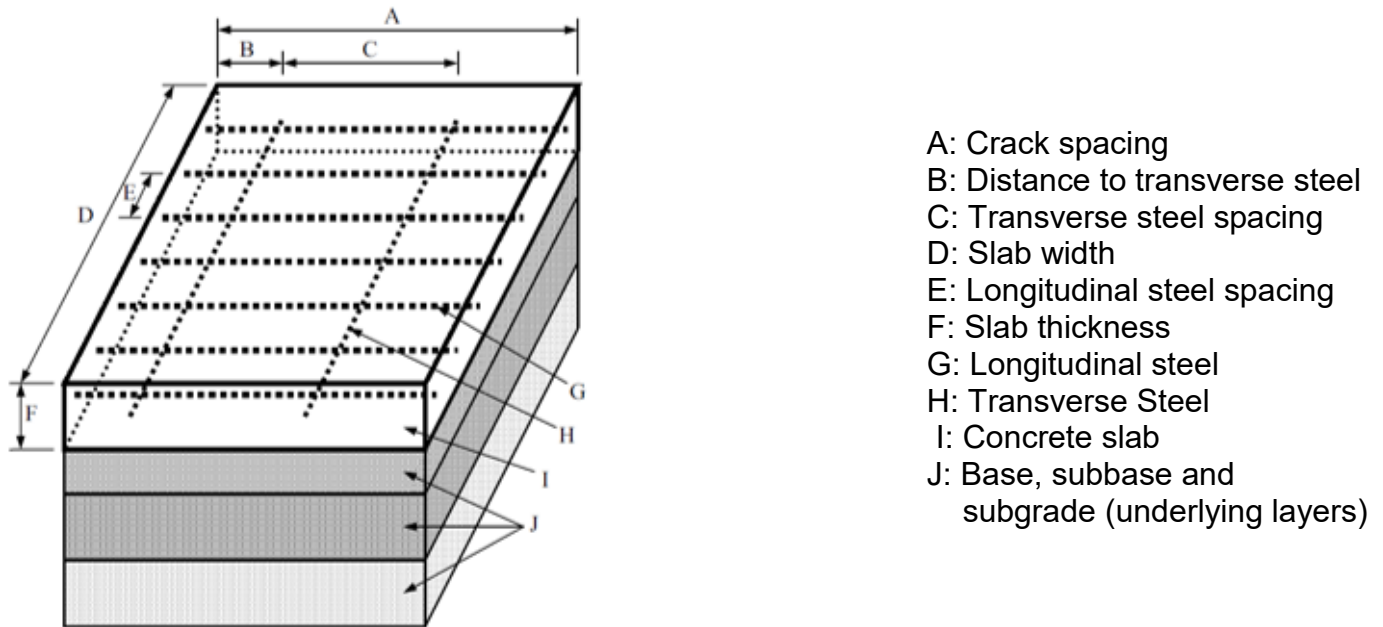
This section describes the functionalities of rigid pavement systems and the concepts available for their design and analysis.

#### **Rigid Pavement System**

Rigid pavement systems consist of a number of portland cement concrete slabs placed over one or more foundation layers (base, subbase and subgrade). In a rigid pavement system, the PCC slab is the stiffest structural element that provides major bearing capacity against the applied loads. Pavement slabs can be composed of layers with different material properties and thickness, with the interface between them considered either bonded or unbonded. The slab layers are usually placed over an unstabilized or stabilized base course, which can also contribute to the load resistance system. However, the main roles of a PCC slab are to provide a uniform support for pavement slabs, contribute to the subgrade drainage and frost protection, improve the foundation strength, and prevent subgrade pumping (Hammons and Ioannides, 1997). One or more sub-base layers may also be used in the pavement foundation system. Sub-bases are usually made with lesser quality granular materials to replace soft and compressible soils. In addition, they can provide strength to the pavement system and offer frost and swelling protection. The last layer in a rigid pavement system is subgrade, which is either natural or compacted soil. The subgrade strength property is represented by resilient modulus, which is dependent on moisture content.

Continuously reinforced concrete pavement consists of continuous longitudinal reinforcement with no transverse expansion or contraction joints except at bridges or pavement ends. The reinforcing steel in CRCP helps ensure that the transverse cracks are tightly held together and, as a result, provide high load transfer over the life of the

pavement. Therefore, an adequate bond-slip model is necessary to represent the unique interaction between the concrete and the reinforcing steel. The structure of a typical CRCP is shown in Figure 1.



**Figure 1. Continuously Reinforced Concrete Pavement Structure (Kim et al., 2000)**

Rigid Pavement Analysis Methods

The mechanical analysis of rigid pavements is based on closed-formulas developed by Westergaard (Westergaard 1927) or, most recently introduced, by the finite element (FE) method. Closed-form formulas are based on several assumptions that may not reflect the analysis of a very large scale slab on liquid foundation. To address the limitations of closed-form formulas, pavement engineers began to implement the use of FE techniques for accurately simulating rigid pavement structures. The FE method has been incorporated into a number of computer software for calculating stresses, strains, and deflections of concrete pavements. The development of FE-based software offers superior accuracy as compared to the original Westergaard closed-form solution. However, the FE-based software also suffer from various limitations related to the complexity of calculations, such as the inability to introduce a yield condition in an

elastic analysis or the modeling of material nonlinearity, which can be difficult to describe mathematically and computationally.

### Rigid Pavement Analysis Tools

The first FE-based tool for the analysis of rigid pavement was developed in 1979 under the ILLI-SLAB software package (Ioannides, 1984). The original FE formulation of that software was based on a 2D plate element, developed by Cheung and Zienkiewicz (Cheung and Zienkiewicz, 1965), on a Winkler foundation (see Figure 2). In ILLI-SLAB, thermal loads could only be considered for one slab with fully bonded or completely unbonded slab-base interface conditions. In addition, only a linear temperature distribution with the slab depth was allowed (Tabatabaie and Barenberg, 1980). Since the first version, ILLI-SLAB has been under continuous revision and verification to improve its accuracy and capability. One of the improvements was the inclusion of elastic solid foundation making ILLI-SLAB the first program to have both types of ideal subgrades (liquid and solid elastic) in one package.



**Figure 2. Slab on Winkler Foundation**

In 1986, Tayabji and Colley developed JSLAB based on ILLI-SLAB formulation. JSLAB had been revised to incorporate partial contact in slab/base interface, to consider non-uniformly spaced dowels in joints, and include the warping effect due to moisture. JSLAB also calculated the thermal and principal stresses (Heinrichs *et al.*, 1989). Continuous improvements to the software resulted in JSLAB2004, which incorporated an axle configuration library and an “Express Mode” interface, while expanding the type of foundation models to six different subgrade types (spring, Winkler, Boussinesq, Vlasov, Kerr, ZSS foundations). JSLAB2004 could analyze jointed concrete pavement responses under self-weight, traffic and thermal loads for a two-layer system of up to nine slabs. The FE model of the slab and the foundation were condensed to just one

layer when thermal loads were applied. For this reason, continuous foundation models (e.g. Vlasov and solid elastic) could not be used in modeling multiple slabs (Carrasco *et al.*, 2011). The program was also able to model the separation between slabs and the foundation caused by positive and negative temperature gradients. However, modeling the horizontal slab-foundation interaction was not possible.

Khazanovich *et al.* developed ISLAB2000, at the ERES Division of Applied Research Associates, which used the Totsky model (Khazanovich, 1994) to analyze interior loading cases more accurately by considering effects of subgrade deformation under slab edges. ISLAB200 offered a variety of subgrade options such as the Pasternak, Kerr and ZSS models. One of the improvements made during ISLAB2000 development was enabling curling analysis of slabs on the Pasternak and Kerr foundations. To do so, it was assumed that the slab and the subgrade were separated as if there was a tensile stress between them. Rewriting of the code improved the software's ability to analyze mismatched joints and cracks, voids, mesh generation, load placement, and batch processing. ISLAB2000 could also solve pavement responses due to temperature, traffic, and construction loading. Moreover, its graphical interface for inputs and outputs made it more user-friendly (Buch *et al.*, 2004).

EverFE is a rigid pavement three-dimensional FE analysis tool which was developed to overcome the limitation of 2D programs. 2D models are not capable of capturing detailed local responses and adequately model shear transfer at joints (Davids *et al.*, 1998). EverFE was able to model up to nine jointed slab-shoulder system. Dowels, tie bars and linear or nonlinear aggregate interlock can be simulated at joints. Dowel looseness, dowel misalignment and misplaced can also be modeled in EverFE's 3D FE model (Davids, 2003). EverFE allowed for specifying up to three either bonded or unbonded elastic base layers. For the unbonded slab-base interface, shear transfer can be captured via a bilinear elastic-plastic curve that defines the shear stresses to relative displacement constitutive relationship. This relationship can be obtained from an experimental push test for each type of base material to define the relative displacement in which slip occurs and to determine the frictional or shear stresses at the slip state. In this method the frictional or shear stresses are independent of the normal stresses. A



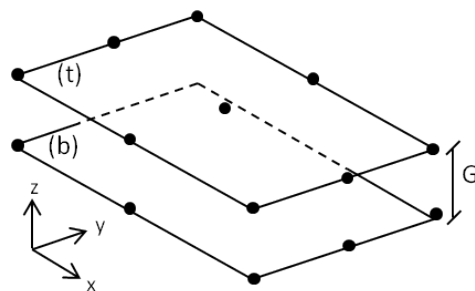
linear or non-linear temperature gradient can be considered in this program. EverFE's finite element code employed 20-node quadratic brick elements to discretize the slab and the elastic base layers; 8-node planar quadratic elements for the dense liquid foundation; and 16-node quadratic interface elements to model both aggregate interlock joint shear transfer and shear transfer at the slab-base interface (Davids, 2003).

In 2004, The National Cooperative Highway Research Program (NCHRP) developed a mechanistic-empirical method for rigid pavement design under project 1-37A, also called the Mechanistic-Empirical Pavement Design Guide (MEPDG) (AASHTO, 2004). The current design procedure for the CRCP pavements consist of two parts; longitudinal reinforcement design and thickness design. Longitudinal reinforcing is considered to control transverse crack spacing, which ASSHTO recommends that allowable crack width should not exceed 0.04 inches and also recommends to limit stress values to the 75 percentile of the ultimate tensile strength. The thickness design procedure for CRCP is the same as the thickness design for JCP, which is based upon an extension of the field performance models developed from the AASHTO Road Test. The approach is based in the empirical relationship between pavement serviceability loss and the magnitude, configuration and repetition of traffic axle loads. This method employed a user-friendly procedure by incorporating several issues, such as actual traffic distribution by using axle load spectra, nonlinear temperature gradient, local environmental condition, local highway materials and damage (crack and faulting) prediction. Many highway agencies have adopted the mechanistic-empirical design guide as a state-of-the-practice tool for the design of new and rehabilitated pavements.

### *Development of NYSLAB*

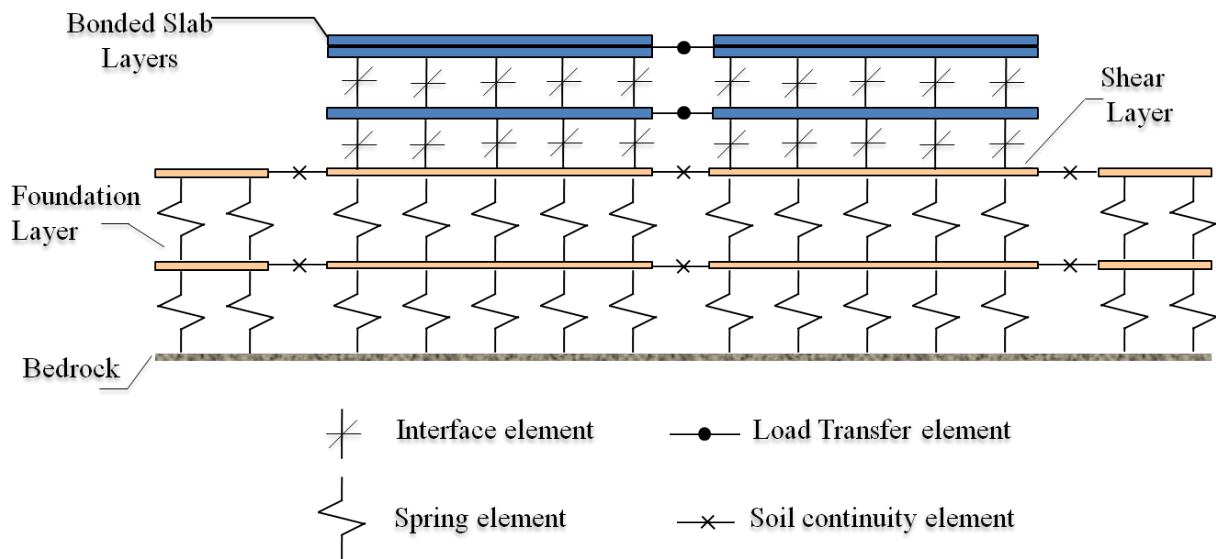
To overcome the limitations of JSLAB2004, researchers from the University of Texas at El Paso developed NYSLAB, coded in Matlab<sup>®</sup>, to calculate responses of jointed concrete pavements for different geometric configurations, foundation models and parameters, and temperature gradient profiles. The software was developed into a standalone executable program with a user-friendly graphical interface. Matlab's built-in capabilities allow the efficient management of matrix and vector operations on which the FE method is based.

Pavement slab layers are modeled as plate elements, with their interface considered either bonded or unbonded. In bonded slabs, shear stresses can be transferred completely in their interface and theoretically no sliding or separation can occur between them. On the other hand, unbonded slabs can move with respect to each other and shear stresses can be produced in their interface when they are subjected to external loads. Bonded pavement slabs are modeled as composite laminated plates in NYSLAB. This allows for the modeling of slab layers with different material properties and thickness with one single composite laminate. The “first order shear deformation laminated plate theory”, or the Mindlin laminated plate theory, is used to model the composite slabs. This model is capable of computing shear deformations, which are important in the modeling of thick slabs. The unbonded slab layers are modeled independently from one another and from the foundation layer. Modeling the contact between these layers is of great importance since the contact conditions along their interface significantly impacts the mechanical behavior of the pavement. For this reason, interface elements were developed to connect unbonded pavement slabs and to connect the bottom slab and the surface of the top foundation layer in order to capture the loss of contact (separation) and slipping between them. An isoparametric 18-node interface element, shown in Figure 3, was developed to model the frictional contact between the slab layers and between the slab and the foundation. This element is compatible with the plate element and the foundation elements used in NYSLAB. The use of this interface element, as opposed to the commonly used 1D spring connections between contacting nodes, allows the proper distribution of the normal and tangential stiffness for non-uniform meshes.



**Figure 3. Interface Element as Modeled in NYSLAB**

Two foundation models are included in NYSLAB to idealize the behavior of the foundation system: Winkler and Vlasov foundation. The Winkler foundation model considers the slab supporting layers as an infinite set of independent linear elastic vertical springs with a constant axial stiffness. All foundation layers (base, subbase and subgrade) are represented as one layer of disconnected spring elements. In this model, the contribution of all foundation layers is manifested as a single modulus of subgrade reaction or  $k$ -value, which is the stiffness of each spring element. The Vlasov foundation model, shown in Figure 4, considers shear interactions between spring elements in each foundation layer and it is more realistic than the simple idealization in the Winkler theory. In this model the layer normal stiffness  $k$  and the layer shear stiffness  $\tau$  of each foundation layer are the two parameters that can be assigned to one or two foundation layers.



**Figure 4. Jointed Pavement Section as Modeled in NYSLAB Using the Vlasov Model**

Uniform temperature-change, within the depth of the pavement slabs during daily temperature variation, can cause thermal expansion and contraction in concrete slabs. The impact of the horizontal interaction between adjacent jointed slabs, due to the additional compressive stress produced by the thermal expansion, is considered in the mathematical model of NYSLAB. The modeling of non-linear thermal gradients is also

available within the program and can be applied to any number of PCC layers (Carrasco *et al.*, 2014).

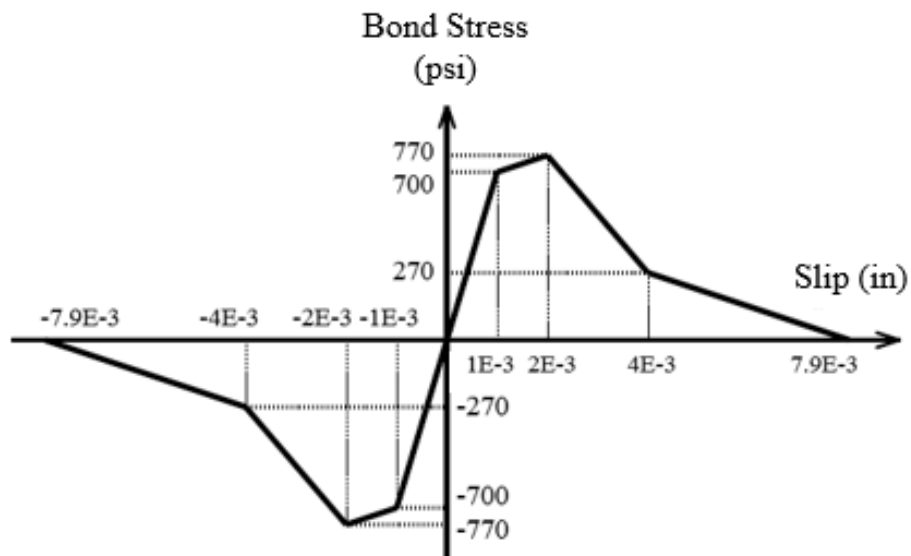
### ***Mechanistic Modeling of CRCP***

The first mechanistic CRCP model was developed in the mid 1970's. Software CRCP-1 was the first computer program with the capability to evaluate the effects of continuous pavement design variables under traffic and environmental loads (Won *et al.*, 1991). A two-dimensional FE model that incorporated the variations in temperature and moisture changes occurring through the depth of the concrete slab was developed for the Texas Department of Transportation in 1996 (Kim *et al.*, 2001). A new software, CRCP-9, was developed in 1998 that incorporated two and three-dimensional FE models to predict the crack spacing using the Monte Carlo simulation method and the failure prediction model using probability theories (Won and McCullough, 2001). Software CRCP-9 analyzes the stresses due to curling and warping of the concrete slab by considering the variations in temperature and drying shrinkage through the depth of the concrete slab. One limitation of CRCP-9 is the calculation of wheel load stresses using the Westergaard equations instead of the FE method. Software CRCP-10 was developed to obtain more realistic wheel load stresses that considered the effect of the moving dynamic tandem axle loads (Won and McCullough, 2001). The dynamic tandem axle loads were calculated by defining relevant variables, such as the load geometry and load time history, and by assuming that the loads are moving, each loaded area is rectangular, and the critical stress is induced by multiple wheels in a tandem axle and their dynamic variations (Kim *et al.*, 2001).

Kim *et al.* compared the strengths and shortcomings of the 2-D and 3-D models used in the CRCP computer programs to determine the most practical model. Modeling in the transverse direction cannot be considered in 2-D models and, therefore, the transverse steel bars and the bond slip between the bars and concrete in the transverse direction cannot be modeled (Kim *et al.*, 2000). The 3-D analysis was performed to validate the accuracy of the 2-D model and to include the modeling of the transverse bars in CRCPs. The analysis on the crack width distribution between the two models determined that the 2-D analysis with the plane stress element underestimated the

crack width from the 3-D analysis. The analysis also determined that the 2-D analysis with the plane strain element overestimated the crack width from the 3-D analysis.

TxCRCP-ME Design Program was developed by researchers from Texas Tech University (TTU) and the University of Texas at Austin (UT), which consists of an Excel spreadsheet that determines CRCP performance (punch-outs per mile) based on user inputs for location, traffic, concrete properties and support layers (Ha *et al.*, 2011). The FE-based mechanistic model used for the development of TxCRCP-ME simulates the behavior of concrete pavements by using three-dimensional solid elements to model the reinforcing steel and the concrete. The interaction between the concrete and the reinforcing steel is considered by modeling the contact area using an 8-node plane quadrilateral interface element. The zero-thickness interface element, which is equivalent to a series of spring elements, was placed between the faces of concrete and the reinforcing steel elements. The interface element represents the relationship between the traction and the relative displacements across the interface and is defined using *Kim et al.* 2000 bond-slip behavior between concrete and reinforcing steel, shown in Figure 5. The supporting layers were modeled using two different models, an elastic-isotropic solid model and the Winkler model. Numerical analyses were conducted using ABAQUS 6.7, an all-purpose FE computer program, to determine the composite modulus of subgrade reaction, composite  $k$ -value, at the top of the base layer. Using the FE program, the base layer was characterized by elastic solid elements and the subgrade was modeled by a set of springs, which have coefficient  $k$ , effective modulus of subgrade reaction. Static pressure was applied on the top surface of the base layer with a 30-in diameter load to measure the average deflection corresponding to the applied pressure load between the center and edge of the loaded area (Ha *et al.*, 2011). The composite  $k$ -value is calculated by dividing the magnitude of the applied pressure load by the average vertical deflections. Finally, the computed composite  $k$  is directly used to determine and evaluate the behavior and performance of the concrete pavement system. A series of static plate load tests were simulated for diverse combinations of support layer properties and were estimated by performing a regression analysis using the SPSS computer program, a software package used for interactive or batched statistical analysis, to account for various foundation combinations.



**Figure 5. Bond Stress-Slip Relation between Concrete and Steel (Kim et al., 2000)**

## **Development of Mathematical Models**

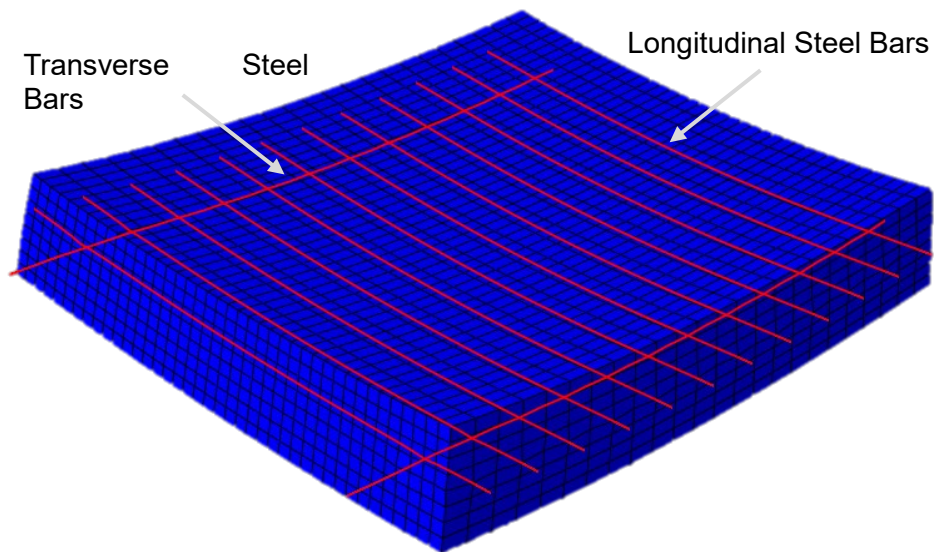
The objective of this task was to develop an independent mathematical model that captures the responses of a section of a CRCP system. This task was address through the following sub-tasks in an effort to accommodate NYSLAB to support the CRCP model.

### ***Asses Modeling Needs***

To first understand the complex interaction between the concrete and the steel, a 3-D FE model was developed in the all-purpose FE software ABAQUS using a model similar to the models incorporated in the CRCP-10 computer program. Using the CRCP 3-D FE model, an analysis was conducted to determine what type of elements, either springs or connectors, were the most appropriate to simulate the connection between the concrete and the steel. The material properties of the CRCP components chosen for the study are provided in Table 1 and the CRCP model created in ABAQUS is shown in Figure 6.

**Table 1. Material Properties of CRCP Model (Kim et al., 2000)**

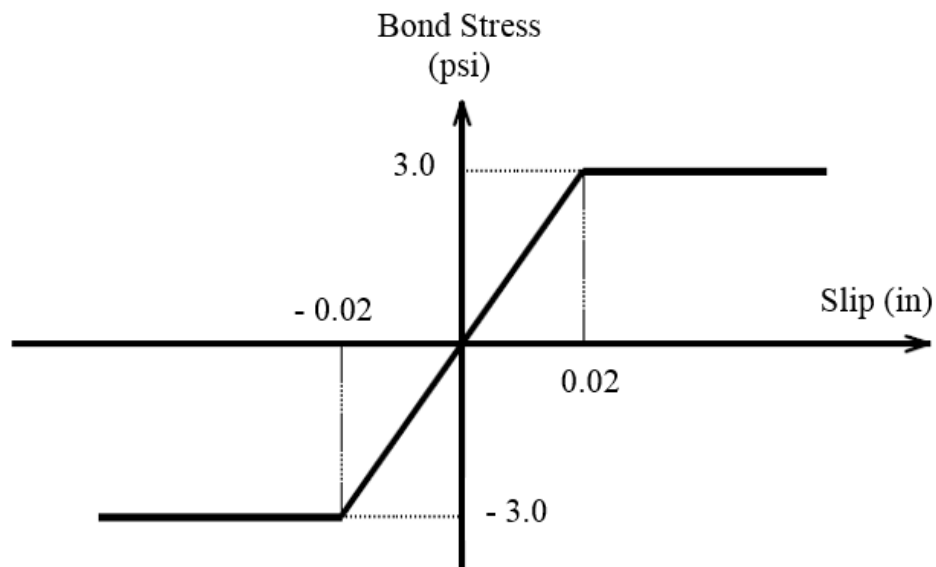
Crack spacing	5 ft	Diameter of transverse steel	0.625 in.
Longitudinal steel spacing	6 in.	Expansion coefficient of concrete	0.000006/°F
Transverse steel spacing	4 ft	Expansion coefficient of steel	0.000005/°F
Concrete slab thickness	12 in.	Surface temperature	85°F
Steel location from surface	6 in.	Bottom temperature	100°F
Concrete modulus of elasticity	4,000,000 psi	Vertical stiffness of underlying layers	400 psi/in.
Poisson's ratio	0.15	Bond slip stiffness between concrete and steel	700 ksi/in.
Diameter of longitudinal steel	0.75 in.	Bond slip stiffness between concrete and base	150 psi/in.



**Figure 6. Three – Dimensional CRCP Model**

The slab was discretized by using three-dimensional brick elements and the reinforcing steel was modeled using beam elements. The underlying layers and the frictional resistance between the concrete and the base were modeled using vertical and horizontal springs, respectively. Kim et al. 2000 reported the bond slip relation between

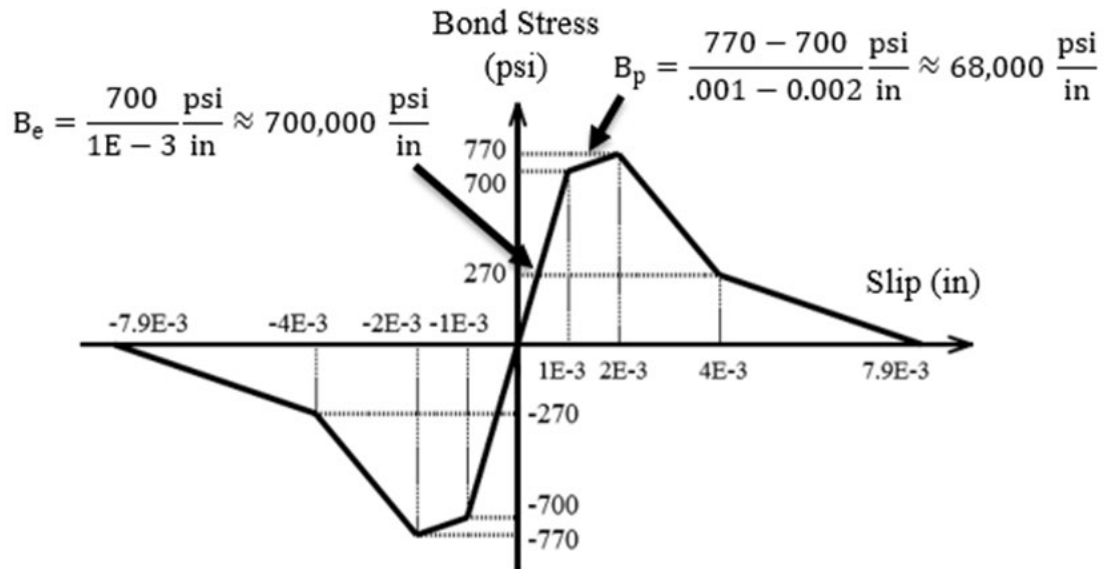
concrete and base as shown in Figure 7. The same relation was incorporated in the model used in this study. The size of each element was selected to be 1.5 in. in the longitudinal and vertical directions and 3 in. in the transverse direction, as suggested by Kim et al. 2000.



**Figure 7. Bond stress-slip relation between concrete and base (Kim et al., 2000)**

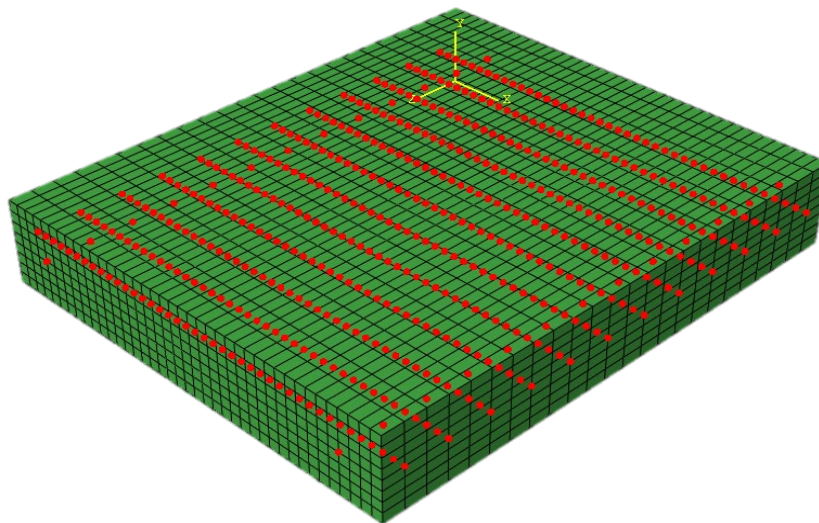
The CRCP-10 computer program uses spring elements to model the bond-slip behavior between concrete and reinforcing steel in both the longitudinal and transverse direction. Figure 8 shows the assumed bond stress-slip relation between the concrete and steel for the 3-D model, where  $B_e$  and  $B_p$  are the elastic and plastic bond slip stiffness values between the concrete and steel, respectively.





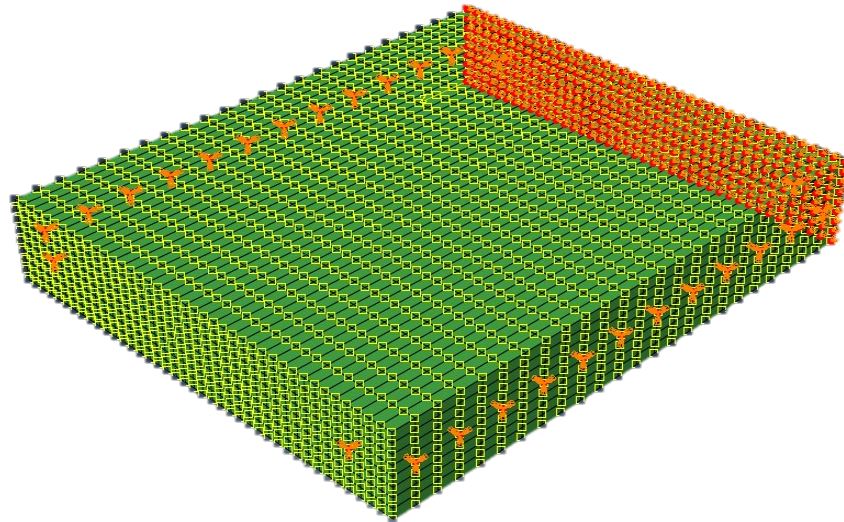
**Figure 8. Bond stress-slip relation between concrete and steel (Kim et al., 2000)**

One of the limitations of using spring elements is that it does not allow the use of advanced material constitutive models (e.g., elastoplastic model) for the bond slip. This can be overcome by modeling the connections between concrete and steel with connector elements. Figure 9 shows the connector elements throughout the slab and their assigned orientation for this study.



**Figure 9. Connector Elements along the CRCP Model**

Figure 10 shows the boundary conditions applied to the CRCP model. The 12-ft-long slab was modeled with cracks placed 5 ft apart. When CRCP is subjected to environmental loading, the response of the pavement system is symmetric with respect to the centerline along the longitudinal direction, therefore, half of the slab, 6 ft, was considered for modeling. In this case, there are no transverse displacements along the symmetric face for concrete and no transverse and rotational displacements for the transverse steels. At cracks, there are no restraints for concrete and no longitudinal and rotational displacements at the longitudinal steel bars. At longitudinal joints, there are no restraints for concrete and no transverse and rotational displacements for the transverse steel. The stress-producing mechanism was a linear temperature variation throughout the depth of the concrete slab.

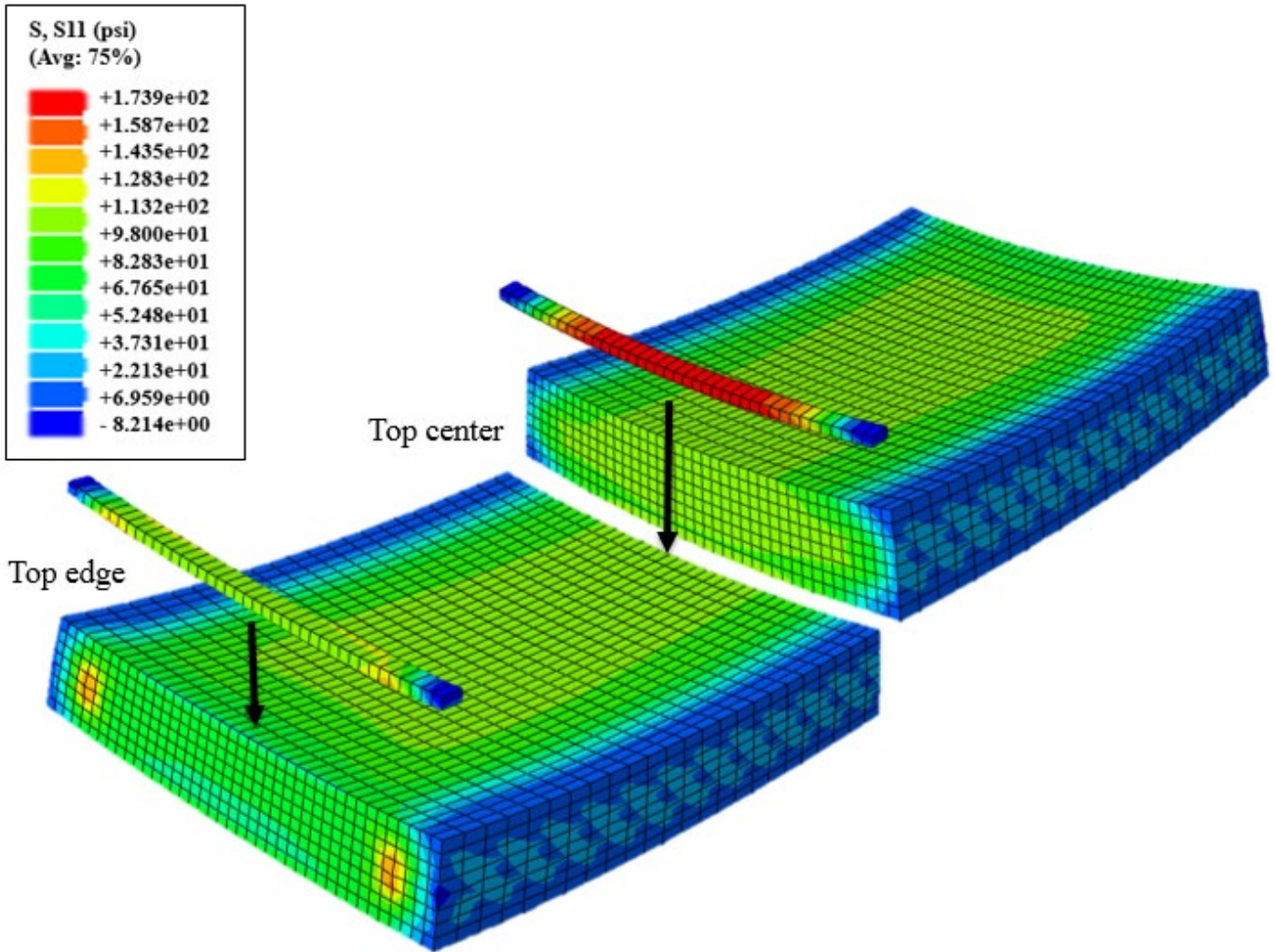


**Figure 10. Boundary Conditions for the CRCP Model**

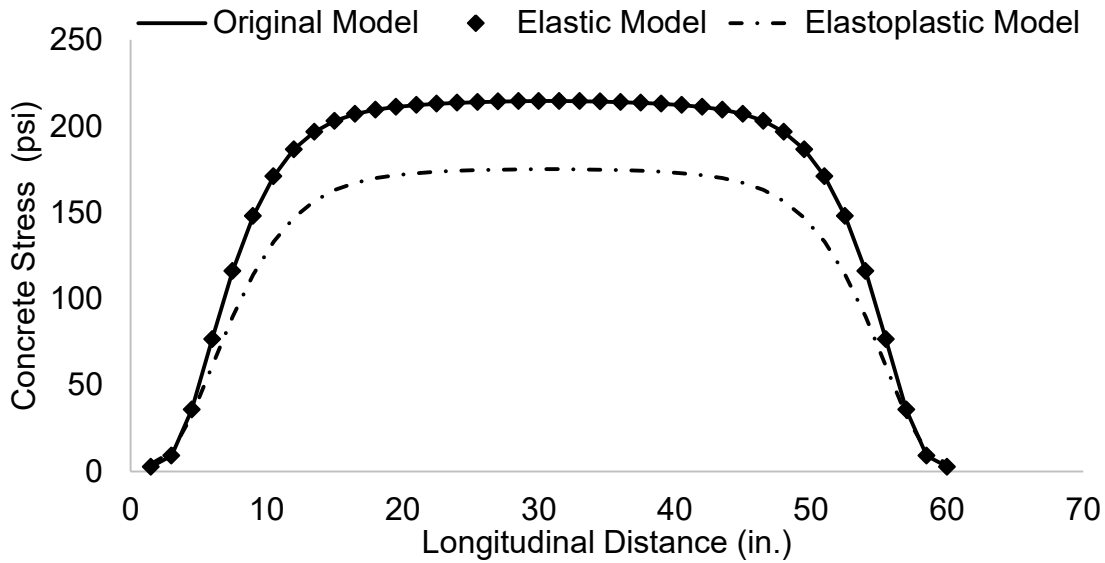
ABAQUS offers a variety of connection models, including connector elements, cohesive elements, mesh-independent point fasteners and mesh-independent surface connections. An attempt was made to model the interaction between the concrete and steel using zero thickness cohesive elements. However, given the nature of the beam elements used to model the reinforcing bar, it was not possible to establish an adequate connection between the brick and beam elements. Mesh-independent surface connections were discarded due to complications recognizing the surfaces of the concrete and steel elements where no connection could be established. Mesh-

independent point fastener combines either connector elements or beam multi-point constraints. Since connector elements can be defined directly and provide more control over the connector definitions, mesh-independent point fasteners were discarded as well. Connector elements CONN3D2 (three-dimensional connector) were considered as the most appropriate to model the interaction between the concrete and steel since a variety of behaviors can be simulated with those connector elements including elasticity, friction, plasticity, damage, and failure. For this study, an elastoplastic behavior using the same linear bond stress-slip relation provided by the previous study was selected for simplicity.

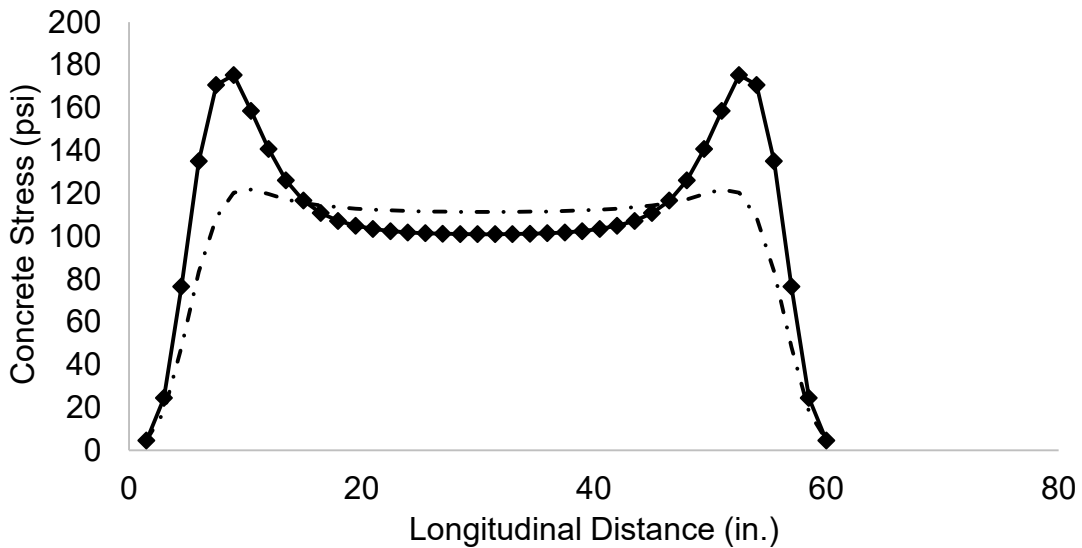
Figure 11 shows the concrete stress distributions along the top center and top edge of the concrete slab along the longitudinal direction obtained using the connector elements to model the interaction between concrete and steel. To verify that the connector elements are accurately modeling the bond slip, a comparison between the original model and the elastic model was performed. As shown in Figure 12, the elastic model matched the original model's concrete stress distributions at both the top center (Figure 12a) and at the top edge (Figure 12b) of the concrete slab. As expected, the concrete stress distributions decreased at the top center and top edge of the concrete slab for the elastoplastic model.



**Figure 11. Concrete stress distribution at the top centerline and top edge of the slab**



(a) Along top centerline

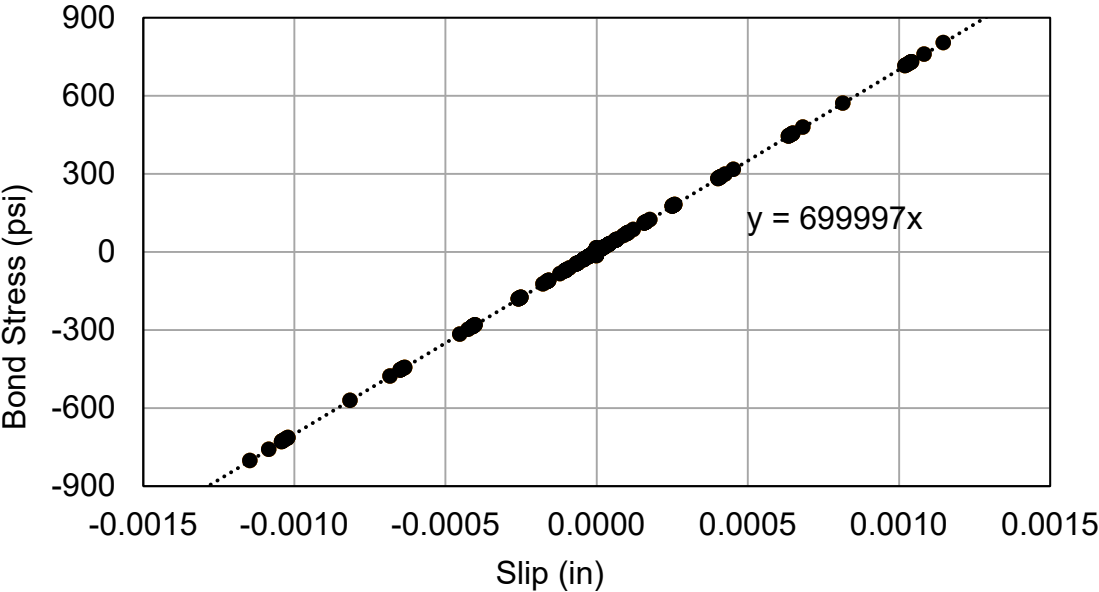


(b) Along top edge

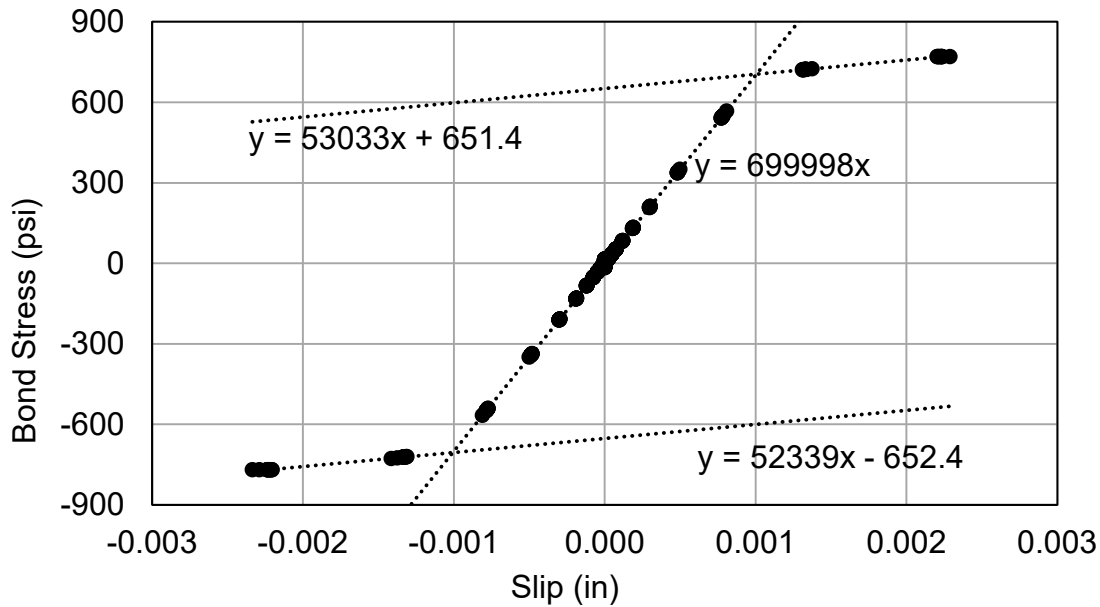
**Figure 12. Concrete stress distribution**

To further verify the accuracy of the connector elements in the CRCP model, the stresses and displacements at the connectors were investigated. Figure 13 shows the bond stress-slip relation for the elastic and the elastoplastic models. The trend line for

the elastic model provides a bond slip stiffness of 699,997 psi/in. compared to the 700,000 psi/in. used in the original model. The trend lines for the elastoplastic model provide an elastic bond slip stiffness,  $B_e$ , between the concrete and steel of 699,998 psi/in. and an elastoplastic bond slip stiffness,  $B_p$ , of 53,033 psi/in., while yield occurs at a bond stress of 711 psi. The bond stress-slip relation input to the model were  $B_e = 700,000$  psi/in. and  $B_p = 68,000$  psi/in., with yield occurring at 700 psi. The connector elements accurately represent the relationship between concrete and steel due to the availability of different material models such as the elastoplastic model. The elastic model does not allow a linear interaction beyond the yield point and consequently provides simplistic results. The elastoplastic model applies actual bond stress-slip data beyond the yield point and applies it to the slab to obtain realistic results beyond the yield point. Because of the success of the connector elements, the mathematical model that governs the elements can be implemented into the main source code of NYSLAB to expand the capabilities of that tool to model the responses of CRCs.



(a) Elastic model



**(b) Elastoplastic model**

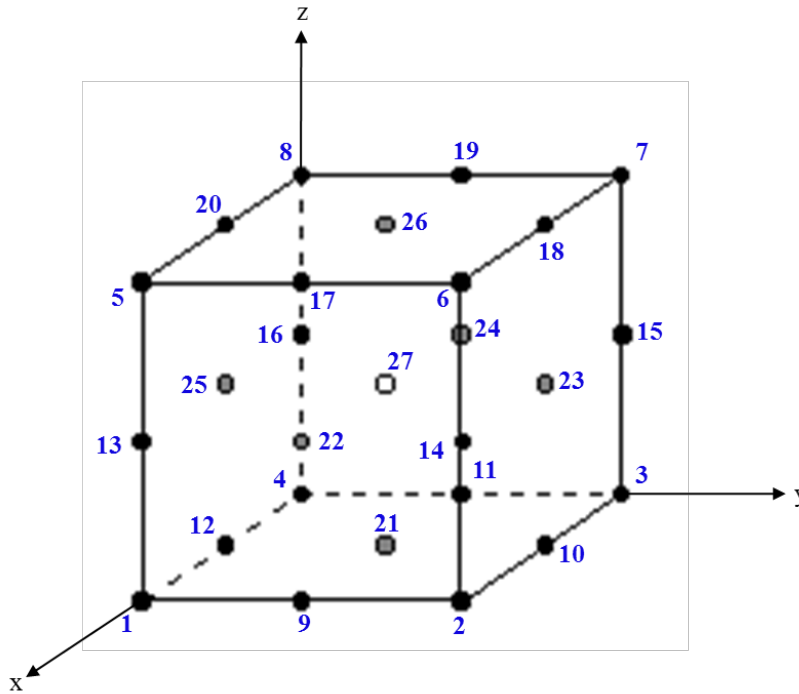
**Figure 13. Bond stress-slip relation for the connector elements**

This study suggested a more rigorous contact element for modeling the interaction between the concrete and steel in CRCP. Comparing the concrete stress distributions at the top of the slab using spring elements with the stresses produced by using connector elements advocates that including an elastoplastic material constitutive model for the bond slip between the concrete and steel provides a more realistic interaction between concrete and steel. Comparing the trend line for the elastic model with the control model, demonstrates that the connector elements accurately represent the interaction between concrete and steel. Analyzing the trend lines for the elastoplastic model with the bond stress-slip relation for concrete and steel shows that the connector elements are functioning according to the assumed behavior.

**Condition NYSLAB to Support Modeling Needs**

The use of a single modulus of subgrade reaction,  $k$ , in NYSLAB limits the true analysis of multiple supporting layers. To overcome this limitation, a 3D Foundation model was developed. The soil was discretized using 27-node hexahedron elements (see Figure 14) for each of the slab’s supporting layers. The use of 27-node elements, as opposed to the typical 8- node or 20-node hexahedron elements, permits a faster computation for

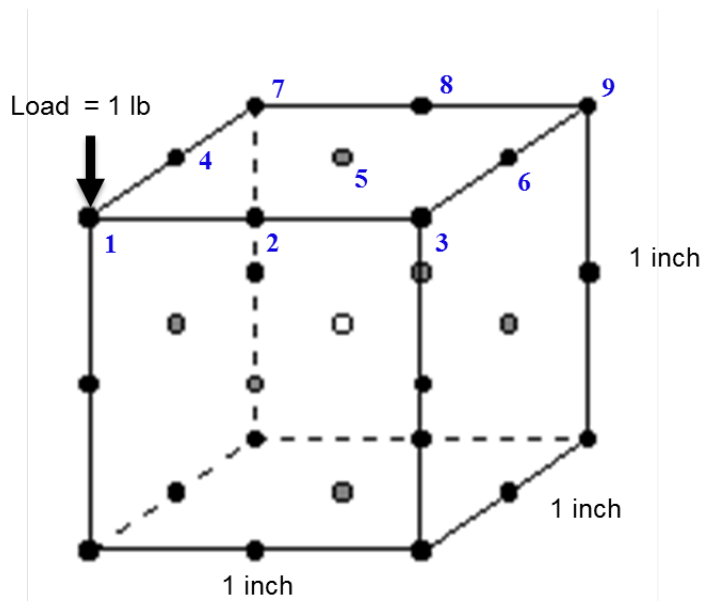
the analysis of pavement systems with multiple supporting layers. This approach will allow engineers to use more familiar layer moduli to represent the foundation layers. Furthermore, this model provides detailed mechanical responses at any depth within the supporting layers.



**Figure 14. 27-Node Hexahedron Element**

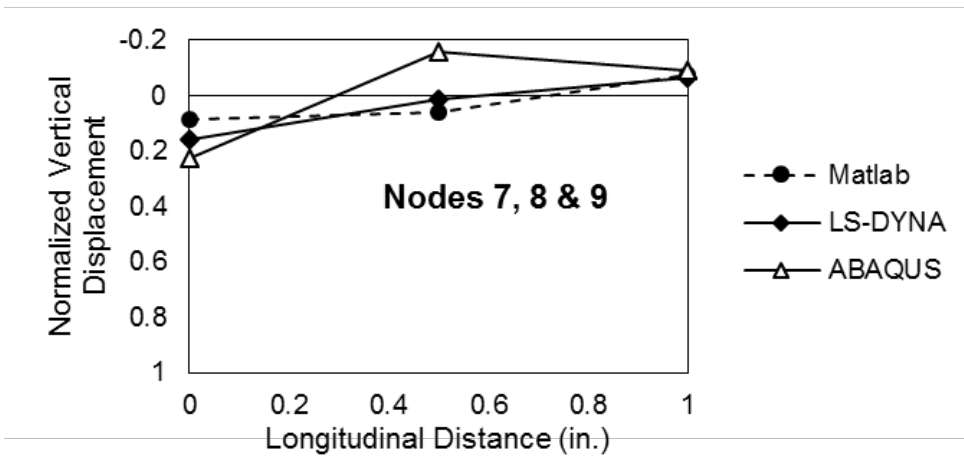
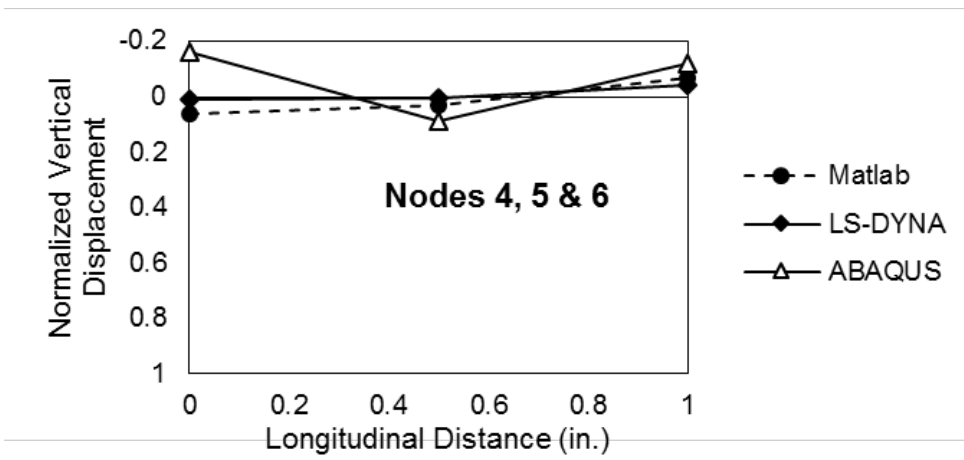
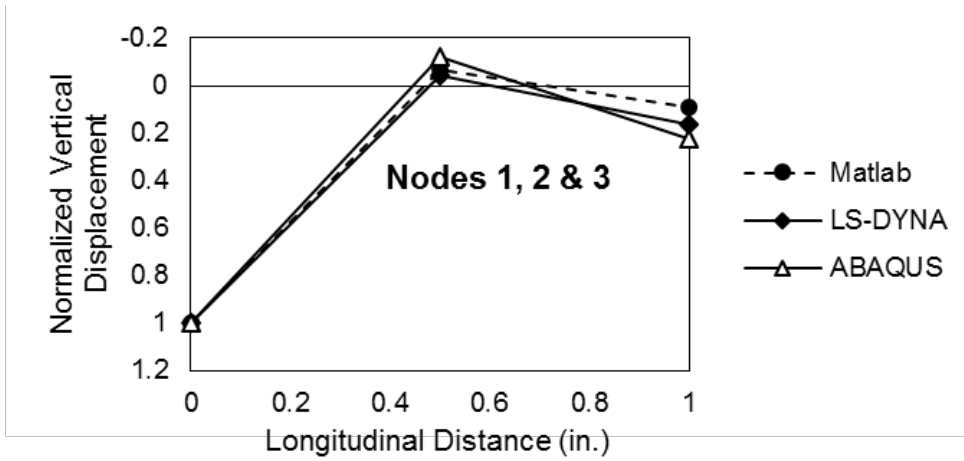
A model was created in MATLAB using the algorithms of the 27 – node hexahedron elements to determine the validity of the proposed element. The model consisted of a unit element fixed at the bottom and subjected to a vertical unit point load as described in Figure 15. The vertical deformations for the top 9 nodes were observed and compared with two similar models created in the all-purpose FE softwares, ABAQUS and LS-DYNA.



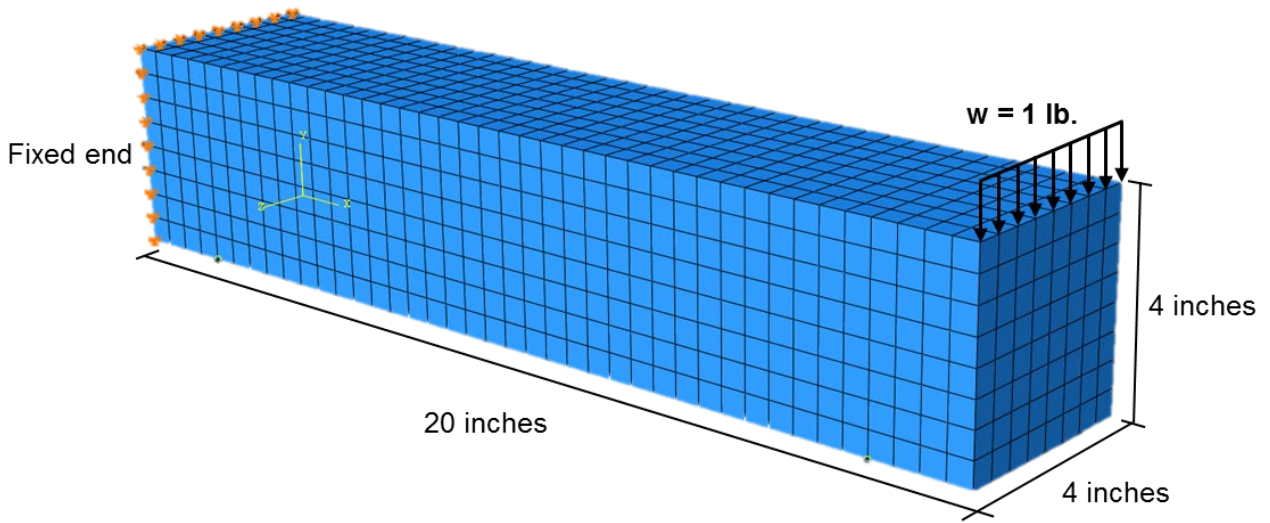


**Figure 15. Unit Element Subjected to a Vertical Point Load**

To better interpret the results obtained, the vertical displacements were normalized with respect to the displacement at node 1. The results for all top nodes are shown in Figure 16. While the results obtained from the LS-DYNA model were very similar to the results from the MATLAB model, the results obtained from the ABAQUS model differed significantly for nodes 4 and 8. Since only one element was used in this study, it was possible that discrepancies occurred as a result of not using an adequate amount of elements to test the model. Therefore, a cantilever beam model, consisting of more elements, was created in MATLAB and compared to a similar model created in ABAQUS (see Figure 17). The 20 in. long beam with a cross sectional area of 4 in. by 4 in. was subjected to a uniform load of 1 pound at the edge of the free end and was assigned a modulus of 500 psi and a Poisson's ratio of 0.35.

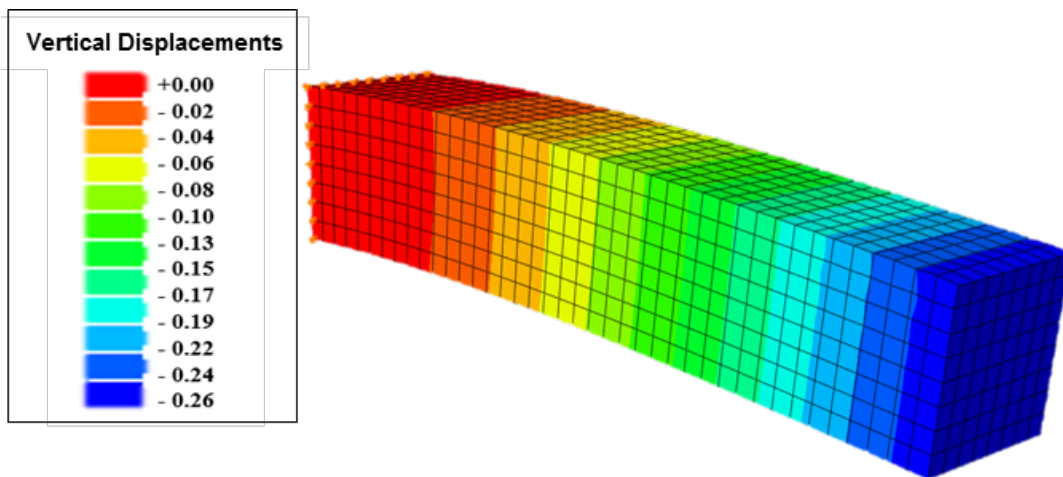


**Figure 16. Normalized Vertical Displacement for Top Nodes**



**Figure 17. Cantilever Beam Model in ABAQUS**

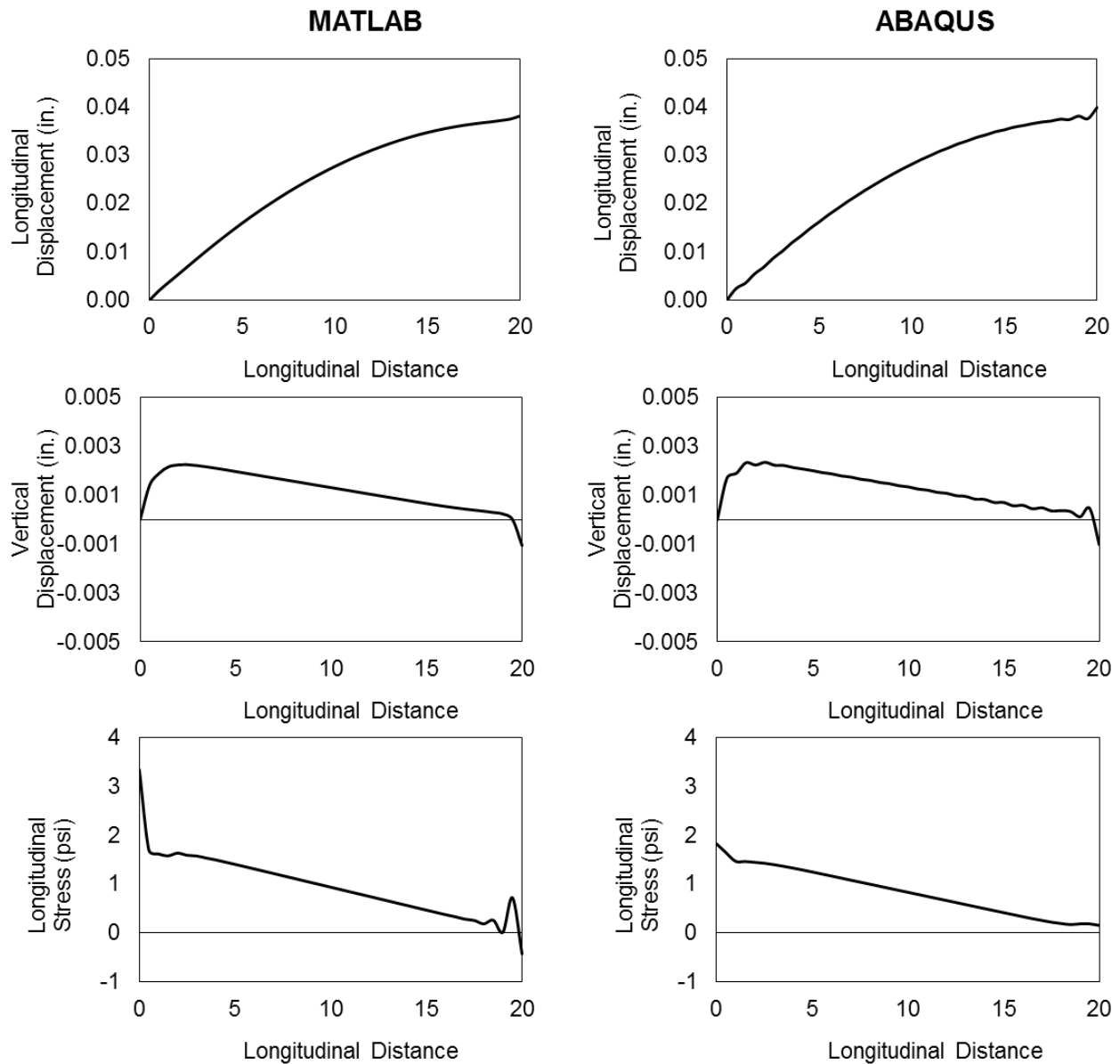
The beam was first solved analytically to determine the maximum vertical displacement using the typical deflection equations of a cantilever beam. The analytical result was compared with the maximum deflections obtained from the MATLAB and ABAQUS model. As shown in Figure 18, the maximum deflections obtained from the cantilever beam model in ABAQUS coordinated with the analytical maximum deflection of 0.25 inches.



**Figure 18. Deflected Cantilever Beam Model in ABAQUS**

Displacements in the x, y and z components from the edge strip of nodes along the longitudinal direction were compared between the model from MATLAB and ABAQUS.

The results from MATLAB also determined a maximum vertical deflection of 0.25 inches and coordinated with deflections obtained from the model in ABAQUS. Figure 19 compares the longitudinal and vertical displacements and the longitudinal stresses obtained from the MATLAB and ABAQUS beam models.



**Figure 19. Comparison of Cantilever Beam Responses**

The mechanical behavior of pavement is significantly influenced by the contact properties as well as frictional characteristics of pavement layers. Thus, an appropriate constitutive relationship for the slab-foundation interface needs to be defined. In addition to the existing contact model in the program that is based on Barbero et al. (1995), a new model has been implemented that associates the normal and frictional properties of each point of contact to the relative displacement of the corresponding nodes in two contacting surfaces (Bhatti 2006). As indicated in Figure 20 (a), the constitutive behavior for the normal contact must satisfy consistency condition, that is the contact force exists only if the gap between two contacting surfaces is closed i.e., either gap  $g$  or force  $F_n$  must be zero. Therefore, the normal constraint function can be approximated by the following equation:

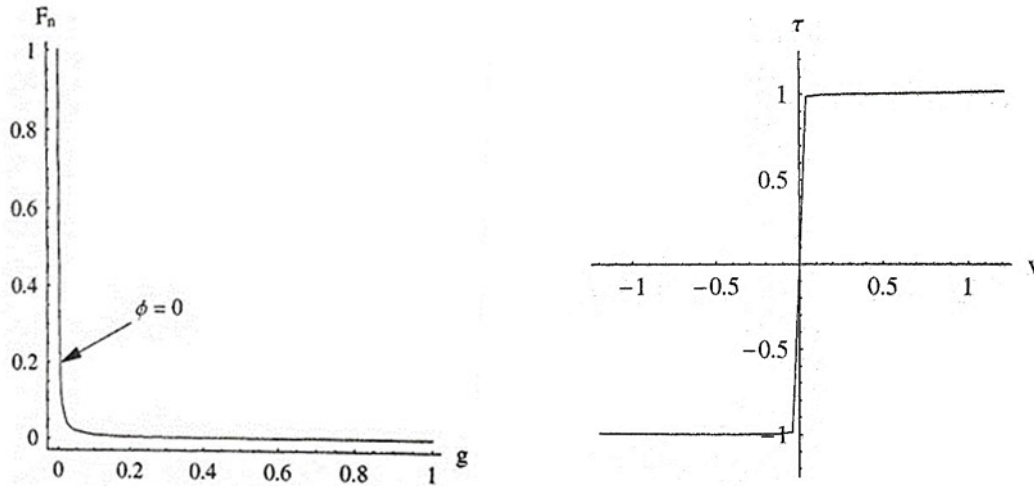
$$\phi(g, F_n) = \frac{g + F_n}{2} - \sqrt{\left(\frac{g - F_n}{2}\right)^2 + \varepsilon} = 0 \quad (1)$$

where  $\varepsilon$  is a small positive number.

Figure 20 (b) shows the frictional properties of the surfaces and the physical condition (i.e., sticking or sliding) play important role. The frictional constitutive equation in the tangential direction, which correlates the frictional tractions  $F_t$  to the tangential relative displacements  $v$ , is expressed as follows:

$$\psi(v, \tau) = \frac{F_t}{\mu F_n} - \frac{2}{\pi} \arctan\left(\frac{v}{\varepsilon}\right) \quad (2)$$

where  $\mu$  is the coefficient of friction between two surfaces.



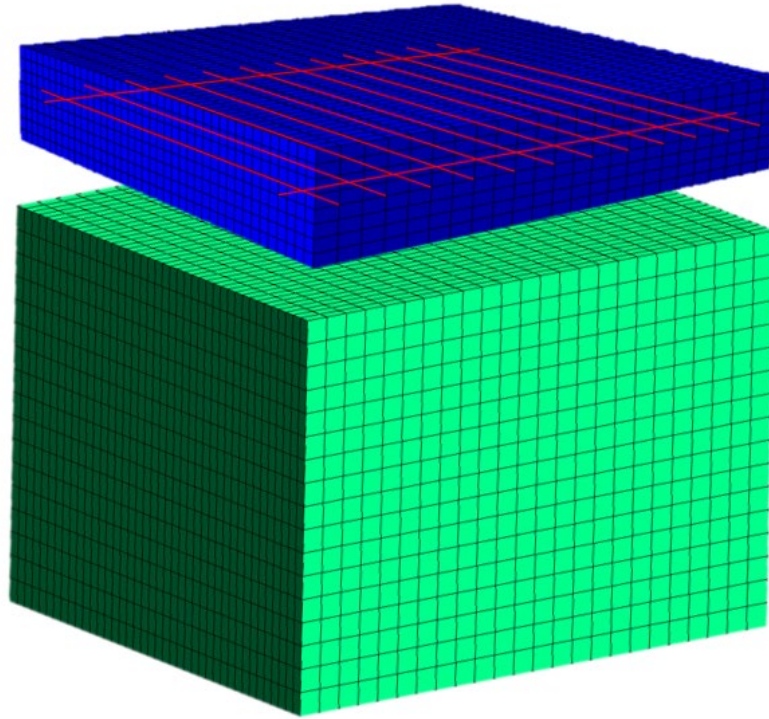
(a) Normal Contact Function      (b) Frictional Constraint Function

**Figure 20. Slab-Foundation Interface Relationship (Bhatti 2006).**

### **Development of Finite Element Model of CRCP**

An illustration of the CRCP model created for this project is shown in Figure 21. The slab was discretized using three-dimensional brick elements and the reinforcing steel was modeled using beam elements. The underlying layers of the concrete slab were modeled using the same brick elements as the concrete slab. The solid elements, as oppose of using spring elements, have the ability to predict pavement responses at any depth within the pavement foundation structure. The frictional resistance between the concrete slab and the base were modeled using the same contact-friction model that was modified in the NYSLAB main source code. The bond-slip behavior between the concrete and the reinforcing steel are modeling using the behavior previously described in the Asses Modeling Needs sub-task, Kim et al., 2000. A nonlinear model that can represent that bond-slip behavior is currently under development. When CRCP is subjected to environmental loading, the response of the pavement system is symmetric with respect to the centerline along the longitudinal direction. In this case, there are no transverse displacements along the symmetric face for the concrete and no transverse and rotational displacements for the transverse steel. At cracks, there are no restraints for concrete and no longitudinal and rotational displacements at the longitudinal steel

bars. At longitudinal joints, there are no restraints for concrete and no transverse and rotational displacements for the transverse steel



***Figure 21. Three Dimensional Representation of the CRCP Model***

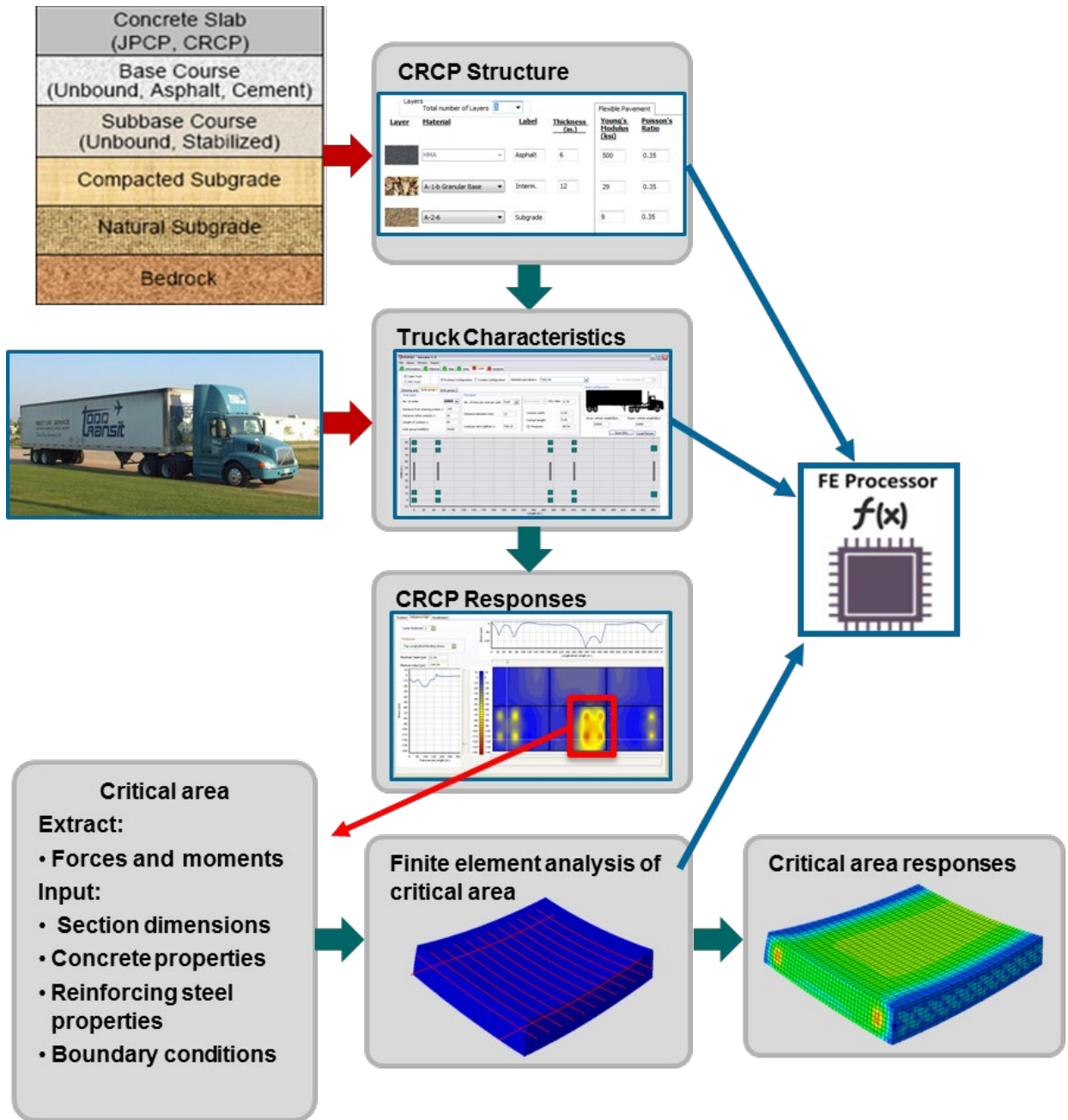
## **Implementation of the CRCP Model**

The mathematical model of CRCP will be implemented into the existing Matlab code using the same modular structure that serves as the foundation of NYSLAB. This modular structure will allow the implementation of individual features of the model in a way that not only enhances the readability of the code but also simplifies its verification by being able to turn on and off different aspects of the model. Figure 22 illustrates the structure of the proposed rigid pavement program for the analysis of continuous pavements.

## **Parametric Studies**

To verify the implementation of the CRCP FE model a series of parametric studies will be developed to study the interplay between the different geometric and material parameters that define the pavement structure. Variations in slab thickness and strength, reinforcing steel configuration, foundation types and traffic and environmental loads will be considered as well as their impact on the response of the pavements will be assessed in terms of stresses, strains and deformations.





**Figure 22. Proposed Structure for the Analysis of Continuously Reinforced Concrete Pavements**

## **Implementation/Technology Transfer (if applicable)**

## References

1. Barbero, E. J., Luciano, R., & Sacco, E. 1995. Three-dimensional plate and contact/friction elements for laminated composite joints. *Computers and Structures*, 54(4), 689-704.
2. Bhatti, M. A. 2006. *Advanced topics in finite element analysis of structures: with Mathematica and MATLAB computations*. New York: John Wiley.
3. Buch, N., D. Gilliland, K. Vongchusiri, and T. J. Van Dam. 2004. *A Preliminary Mechanistic Evaluation of PCC Cross-Sections using ISLAB2000—A Parametric Study*.
4. Carrasco, C., M. Limouee, C. Tirado, S. Nazarian, and J. Bandaña. 2011. *Verification of NYSLAB a Software for the Analysis of Jointed Pavements*. The University of Texas at El Paso, El Paso, Texas.
5. Carrasco, C. J., M. A. Zokaei, C. Tirado, and S. Nazarian. 2014. *Enhanced Finite Element Modeling of the Thermo-Mechanical Responses of Jointed PCC Pavements under Environmental and Traffic*. The University of Texas at El Paso, El Paso, Texas.
6. Cheung, Y. and O. Zienkiewicz. 1965. Plates and Tanks on Elastic foundations - an Application of Finite Element Method. *International Journal of Solids and Structures*, 1 (4): 451-461.
7. Caltrans. 2015. *Concrete Pavement Guide*. Division of Maintenance Pavement Program, State of California Department of Transportation.
8. Davids, Bill. 2003. *EverFE Theory Manual*.
9. Davids, W. G., G. M. Turkiyyah, and J. P. Mahoney. 1998. *EverFE: Rigid Pavement Three - Dimensional Finite Element Analysis Tool*. *Transportation Research Record: Journal of the Transportation Research Board*, 1629 (1): 41-49.
10. *Guide for Mechanistic-Empirical Design of New and Rehabilitated Pavement Structures*. 2004: National Cooperative Highway Research Program, Transportation Research Board, I-37A.

11. Ha S., Yeon J., Choi B., Jung Y., Zollinger D. G., Wimsatt A., Won M. C. 2011. Develop Mechanistic-Empirical Design for CRCP. Report FHWA/TX-11-0-5832-1, Texas Tech University, Lubbock, Texas.
12. Hammons, M. I. and A. M. Ioannides. 1997. Advanced Pavement Design: Finite Element Modeling for Rigid Pavement Joints. Report 1: Background Investigation.
13. Heinrichs, K. W., M. J. Liu, M. I. Darter, S. H. Carpenter, and A. M. Ioannides. 1989. Rigid Pavement Analysis and Design. Final Report. Phase I. Report Number: FHWA-RD-88-068.
14. Ioannides, A. M., J. Donnelly, M. Thompson, and E. Barenberg. 1984. Analysis of Slabs-on-Grade for a Variety of Loading and Support Conditions. Annual Report, 1. University of Illinois at Urbana-Champaign.
15. Khazanovich, L. 1994. Structural Analysis of Multi-Layered Concrete Pavement Systems. PhD Dissertation. University of Illinois at Urbana-Champaign.
16. Khazanovich, L. 2003. Finite Element Analysis of Curling of Slabs on Pasternak Foundation. 16<sup>th</sup> ASCE Engineering Mechanics Conference, Seattle, WA.
17. Khazanovich, L. and A. M. Ioannides. 1993. Finite Element Analysis of Slabs-on-Grade using Higher Order Subgrade Soil Models. Proceeding of the Airport Pavement Innovations Conference. Theory to Practice, Vicksburg, Mississippi, USA.
18. Khazanovich, L. and A. M. Ioannides. 1994. Structural Analysis of Unbonded Concrete Overlays under Wheel and Environmental Loads. Transportation Research Record: Journal of the Transportation Research Board, 1449 (1): 174-181.
19. Khazanovich, L., H. Yu, S. Rao, K. Galasova, E. Shats, and R. Jones. 2000. ISLAB2000 – Finite Element Analysis Program for Rigid and Composite Pavements. User's Guide. Champaign, IL: ERES Consultants.
20. Khazanovich, L., S. D. Tayabji, and M. I. Darter. 2001. Backcalculation of Layer Parameters for LTPP Test Sections, Volume 1: Slab on Elastic Solid and Slab on Dense-Liquid Foundation Analysis of Rigid Pavements.

21. Kim, S., M. Won, and B.F. McCullough. 1998. Numerical Modeling of Continuously Reinforced Concrete Pavement Subjected to Environmental Loads. Paper No. 98-1070, Transportation Research Record 1629, TRB, National Research Council, Washington, D.C.
22. Kim, S., M. Won, and B.F. McCullough. 2000. Three – dimensional Nonlinear Finite Element Analysis of Continuously Reinforced Concrete Pavements. Report FHWA/TX-00/1831-1, Center for Transportation Research. The University of Texas at Austin, Austin, Texas.
23. Kim, S., M. Won, and B.F. McCullough. McCullough. 2001. CRCP – 9: Improved Computer Program for Mechanistic Analysis of Continuously Reinforced Concrete Pavements. Project 0-1831-2. FHWA, U.S. Department of Transportation.
24. Kim, S., M. Won, B.F. McCullough, and R. River. 2001. CRCP – 10 Computer Program User's Guide. Publication FHWA/TX-0-1831-4. FHWA, U.S. Department of Transportation.
25. Mechanistic-Empirical Pavement Design Guide: A Manual of Practice, AASHTO, 2008.
26. McCullough, B. F., and T. T. Office. 1994. Analysis of Jointed Concrete Pavement. University of Texas at Austin, Austin, Texas.
27. National Research Council. 1979. Failure and Repair of Continuously Reinforced Concrete Pavements. Report No. 60, Highway Research Board.
28. Pasko, T. J. 1998. Concrete Pavements – Past, Present, and Future. Public Roads, Vol. 62, 1998.
29. Plei, M., and S. Tayabji. 2012. Continuously Reinforced Concrete Pavement Performance and Best Practices. Publication FHWA-HIF-12-039. FHWA, U.S. Department of Transportation.
30. Roesler, J. R., Hiller, J. E., Brand, A. S. 2016. Continuously Reinforced Concrete Pavement Manual: Guidelines for Design, Construction, Maintenance, and Rehabilitation. Report FHWA-HIF-16-026, Federal Highway Administration.
31. Tabatabaie, A. M. and E. J. Barenberg. 1980. Structural Analysis of Concrete Pavement Systems. Transportation Engineering Journal, 106 (5): 493-506.

32. Tayabji, S. D. and B. E. Colley. 1986. Analysis of Jointed Concrete Pavements.
33. Tayabji, S. D. and B. E. Colley. 1983. Improved Rigid Pavement Joints.  
Transportation Research Record: Journal of the Transportation Research Board, 930 (1): 69-78.
34. Won, M. 2009. Evaluation of MEPDG with TxDOT Rigid Pavement Database. Report FHWA/TX-09/0-5445-3 Center for Transportation Research. The University of Texas at Austin, Austin, Texas.
35. Won, M., and B.F. McCullough. 2001. Mechanistic Analysis of Continuously Reinforced Concrete Pavements. Research Project Summary Report 1831-S. Texas Department of Transportation.
36. Won, M.; Hankins, K.; and McCullough, B. F. 1991. Mechanistic Analysis of Continuously Reinforced Concrete Pavements Considering Material Characteristics, Variability, and Fatigue. Research Report 1169-2, Center for Transportation Research, University of Texas at Austin, Austin, Texas.
37. Zollinger, D., N. Buch, D. Xin, and J. Soares. 1999. Performance of CRC Pavements Volume VI - CRC Pavement Design, Construction, and Performance. Publication FHWA-RD-97-151. FHWA, U.S. Department of Transportation.